

Higher Derivative Terms in the Effective Action of N=2 SUSY QCD from Instantons.

Alexei Yung ^{*}

Petersburg Nuclear Physics Institute
Gatchina, St. Petersburg 188350, Russia

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Abstract

We consider N=2 SUSY QCD with gauge group SU(2) and N_f flavours of matter with nonzero mass. Using the method of the instanton-induced effective vertex we calculate higher derivative corrections to the Seiberg-Witten result in the momentum expansion of the low energy effective Lagrangian in various regions of the modular space. Then we focus on a certain higher derivative operator on the Higgs branch. We show that the singular behavior of this operator comes from values of mass of matter at which charge singularity on the Coulomb branch collides with the monopole or dyon one. Given the behavior of this operator at weak coupling coming from instantons as well as its behavior near points of colliding singularities we find the exact solution for this operator.

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^{*}e-mail address: yung@thd.pnpi.spb.ru

1 Introduction

Last few years a considerable progress has been made in the understanding of the strongly coupled dynamics of supersymmetric gauge theories in four dimensions using the idea of the electromagnetic duality [1]. It was initiated in the celebrated papers by Seiberg and Witten [2, 3] where exact prepotentials on the Coulomb branch has been obtained for $N=2$ supersymmetric $SU(2)$ gauge theories with and without matter hypermultiplets. Later on exact prepotentials has been obtained in $N=2$ supersymmetric gauge theories with various gauge groups [4] and various matter hypermultiplets content [5] (see also [6] for a review of recent results in $N=1$ supersymmetry).

However, it is not quite clear to what extent $N=2$ theories are actually exactly soluble. The exact formula for masses of BPS states [2, 3] indicates that some integrable structure could persist for states with nonzero masses as well.

In particular, in this paper we address the issue of higher derivative corrections in the momentum expansion of the low energy effective theory. From this point of view the Seiberg-Witten solution [2, 3] represent just the first term in this expansion (presumably infinite).

To approach this problem we use instanton calculations in the microscopic theory. First principle microscopic instanton calculations prove to be a powerful method for the testing of proposed exact solutions for prepotentials in $N=2$ SUSY gauge theories. Each term in the expansion of the exact prepotential in powers of the scale parameter Λ of the gauge theory should coincides with the result coming from instanton with a given topological charge. These tests resulted in the agreement for $SU(2)$ gauge theory with $N_f = 0, 1, 2$ flavours of matter at the one-instanton level [7] as well as at two-instanton level [8], while for $N_f = 3, 4$ some discrepancies have been reported [9]. Theories with $SU(N_c)$ gauge group have been also studied [10].

A somewhat different instanton approach has been taken in our paper [11]. All effects produced by instanton in the microscopic theory at large distances can be encoded in the certain local effective vertex. For gauge theories this vertex has been suggested a long ago by Callan, Dashen and Gross [12] (see also [13]). In ref. [11] the similar vertex has been found for $N=2$ pure gauge theory. Restricted to light degrees of freedom this vertex produces all terms in the momentum expansion of the low energy effective Lagrangian at the one-instanton level. The leading term in this expansion coincides with

the one-instanton term of Seiberg-Witten exact solution for the prepotential while others represents all orders of higher derivative corrections [11].

In this paper we continue to study instanton-induced vertices in $N=2$ SUSY gauge theories. We consider $SU(2)$ gauge theory with $N_f = 1, 2, 3$ hypermultiplets of matter in the fundamental representation of the gauge group.

The modular space of this theory consists of the Coulomb branch as well as Higgs branches [3]. The vacuum expectation value of matter field is zero $\langle Q \rangle = 0$, while $\langle \Phi^a \rangle = \delta^{3a}$ $\langle A \rangle \neq 0$ on the Coulomb branch. Here we use $N=1$ superfield notations Q^{kA} for matter fields (k is the colour index $k = 1, 2$, and $A = 1, \dots, N_f$ is the flavour index) and Φ^a ($a = 1, 2, 3$) for the adjoint chiral $N=1$ superfield which is a part of $N=2$ vector multiplet. Hence, the gauge group is broken to $U(1)$ on the Coloumb branch, the light fields being the photon and its superpartners.

The points of special interest on the Coloumb branch are the singular points, where some monopoles, dyons or charges become massless [2, 3]. The prepotential $\mathcal{F}_{N_f}(A)$ has logarithmic singularities at these points coming from contributions of massless particles to the effective coupling constant

$$\tau = i \frac{4\pi}{g_{eff}^2} + \frac{\theta}{2\pi} = 4\pi i \frac{\partial^2 \mathcal{F}_{N_f}(A)}{\partial A^2}. \quad (1.1)$$

From the point of view of semiclassical instanton calculations we are particularly interested in the charge singularity that appears in the weak coupling region of the Coulomb branch provided the mass of matter is large $|m| \gg \Lambda_{N_f}$ [3]. The appearance of this singularity can be understood as follows. Due to the presence of Yukawa term in the superpotential $\sqrt{2}\bar{Q}\Phi Q$ some of the matter fields become massless at $\langle A \rangle = -\sqrt{2}m$ because of the cancellation between Yukawa term and the mass term. Once, mass is large $|m| \gg \Lambda_{N_f}$, this singularity appears in the weak coupling region of the Coulomb branch at large $\langle A \rangle$. We consider masses of matter hypermultiplet to be equal to preserve $SU(N_f)$ global symmetry. If $N_f > 1$, the Higgs branch develops with $\langle Q \rangle \neq 0$, which touches the Coulomb branch at the point of singularity $\langle A \rangle = -\sqrt{2}m$ [3].

In this paper we use the instanton-induced vertex method to calculate one-instanton contributions to the momentum expansion of the low energy effective Lagrangian in three different regions of the modular space. First

one is at large $\langle A \rangle$ far away from the singularity at $\langle A \rangle = -\sqrt{2}m$. We study this region as a check of our method. At large m we recover the same result as in pure gauge theory without matter with the scale of gauge theory given by RG-flow condition¹

$$\Lambda_o^4 = m^{N_f} \Lambda_{N_f}^{4-N_f} \quad (1.2)$$

Second is the region of the Coulomb branch close to the singularity at $\langle A \rangle = -\sqrt{2}m$. We work out the effective Lagrangian near this point. It depends on both the massless U(1) gauge field as well as on the light matter field.

The third region is the Higgs branch. The Higgs branch is a hyper-Kahler manifold and admits neither perturbative nor instanton corrections to the metric [3]. However, we show that it gets a non-zero contribution from a certain higher derivative operator of light matter fields. The coefficient in front of this operator depends on mass as

$$\frac{\Lambda_{N_f}^{4-N_f}}{m^4} \quad (1.3)$$

We argue that in strong coupling region of $|m| \sim \Lambda_{N_f}$ this coefficient is given by an analytic function $Y(m)$ which can receive only multi-instanton corrections

$$Y(m) = \frac{1}{m^{N_f}} \sum_{k=0}^{\infty} J_k \left(\frac{\Lambda_{N_f}}{m} \right)^{(4-N_f)k} \quad (1.4)$$

Then we focus on the most interesting case $N_f = 2$. Giving the behaviour of the function $Y(m)$ at large m (1.3) we find the exact solution to this function at any m . As an input we use the conjecture that the only singularities of $Y(m)$ at $m \sim \Lambda_2$ are those related to the singularities of the prepotential $\mathcal{F}_2(A)$.

Namely, we consider particular values of m at which the charge singularity (the root of the Higgs branch under consideration) collide with monopole or dyon singularities. These colliding singularities have been studied in [14].

We estimate the behaviour of $Y(m)$ near these two points of colliding singularities at $m \sim \Lambda_2$ and present the exact solution for $Y(m)$ which satisfy all the necessary conditions.

¹In this paper we use Pauli-Villars regularization scheme which ensures RG-flow condition (1.2) [7].

The paper is organized as follows. In Sect. 2 we review the instanton-induced vertex for the pure $N=2$ gauge theory. In Sect. 3 we find this vertex for the theory with matter. In Sect. 4 we integrate over collective coordinates of instanton and obtain the effective Lagrangian for light fields in various regions of the modular space. In Sect. 5 we find the exact solution for the function $Y(m)$. Sect. 6 contains our conclusive remarks.

2 Instanton-induced vertex in $N=2$ Yang-Mills theory.

In this section we review briefly the instanton-induced vertex method for $N=2$ $SU(2)$ theory without matter [11].

Consider first the non-supersymmetric gauge theory with Higgs fields ϕ^a in the adjoint representation of the gauge group. Then at large distances $(x - x_0)^2 \gg \rho^2$ (x_0 is the position of the instanton center, while ρ is its size) instanton can be represented in the framework of perturbation theory as a point-like vertex. This vertex has the form [12, 13, 15]

$$V^{GH} = -c \int d^4x_0 \frac{d\rho}{\rho^5} (\rho \Lambda)^b \frac{d^3u}{2\pi^2} \exp\left[-\frac{4\pi^2}{g^2} \rho^2 \bar{\phi}^a \phi^a + \frac{\pi^2}{g^2} i \rho^2 \text{Tr}(\bar{\sigma}_\mu \sigma_\nu \bar{u} \tau^a u) F_{\mu\nu}^a\right], \quad (2.1)$$

where u is the orientation matrix $u^{\alpha\dot{\alpha}} = u_\mu \sigma_\mu^{\alpha\dot{\alpha}}$ ($u_\mu^2 = 1$), while b is the first coefficient of the β -function. Vertex (2.1) generates all correlation functions of type

$$\langle A_\mu(x_1), \dots, A_\nu(x_n), \phi(y_1), \dots, \phi(y_k) \rangle \quad (2.2)$$

in the instanton background. To see this let us calculate the correlation function (2.2) in the instanton background. To the leading order in coupling constant g^2 it is given by the product of the classical instanton gauge fields

$$A_\mu(x) = \eta_{\mu\nu}^a \rho^2 \frac{u \tau^a \bar{u}}{(x - x_0)^4 H} (x - x_0)_\nu, \quad (2.3)$$

and scalar fields [16]

$$\phi(x) = \frac{\tau^3}{2} \frac{\langle a \rangle}{H} \quad (2.4)$$

where

$$H = 1 + \frac{\rho^2}{(x - x_0)^2} \quad (2.5)$$

and $\langle a \rangle$ is the VEV of ϕ^a . We use matrix notation for $A_\mu = \tau^a/2 A_\mu^a$ and $\phi = \tau^a/2 \phi^a$ in (2.3), (2.4). In the large distance limit we can ignore function H in (2.3) and keep only the first nontrivial term of its expansion in power of $\rho^2/(x - x_0)^2$ in (2.4).

Now it is easy to see that the same result for the correlation function (2.2) can be obtained on the purely perturbative grounds inserting the vertex (2.1) in the action and calculating the graph with n gauge boson external legs and k scalar external legs to the leading order in g^2 .

Note, that to derive (2.1) we assume that the theory is in the Higgs phase and the VEV $\langle a \rangle \neq 0$ is developed. The reason for this assumption is that we can control only terms which are linear in quantum fluctuations in the exponent in (2.1) in our approximation [11]. However, it is clear that the effective vertex (2.1) can depend only on the whole field ϕ^a rather than on its VEV. Therefore, from now on we use effective vertex (2.1) (and similar vertices below) no matter if VEV is developed or not.

Let us consider now the $N=2$ supersymmetrization of (2.1). It has the form [11]

$$V_{YM} = -\frac{\Lambda_0^4}{4\pi^2} \int d^4x \frac{d\rho}{\rho} \frac{d^3u}{2\pi^2} d^2\theta^1 d^2\theta^2 d^2\bar{\theta}_1 d^2\bar{\theta}_2 \frac{1}{\Psi^a \bar{\Psi}^a} \exp \left[-\frac{4\pi^2}{g^2} \rho_{inv}^2 \bar{\Psi}^a \Psi^a - \frac{\pi^2}{\sqrt{2}g^2} \rho_{inv}^2 i(\bar{\nabla}_f \bar{u} \tau^a u \bar{\nabla}^f) \bar{\Psi}^a \right] \quad (2.6)$$

Here $\Psi^a(x, \theta^1, \theta^2)$ is the $N=2$ chiral superfield [17]. Its lower components can be written in the form (for a recent review see [18])

$$\Psi^a(x, \theta^1, \theta^2) = \Phi^a(x, \theta^1) + \sqrt{2}\theta_{2\alpha} W^{\alpha a}(x, \theta^1) + \dots \quad (2.7)$$

where Φ^a is the scalar $N=1$ superfield while $W^{\alpha a}$ is the field strength of the $N=1$ vector superfield. $N=2$ superderivatives ∇_f^α , $\bar{\nabla}_{\dot{\alpha}}^f$ ($\alpha, \dot{\alpha} = 1, 2$ is the spinor index, while $f = 1, 2$ is the $SU_R(2)$ index which counts the first and the second supersymmetry) satisfy the anticommutation relations (see, for example, review [19])

$$\begin{aligned} \{\nabla_f^\alpha, \nabla_p^\beta\} &= \epsilon^{\alpha\beta} \epsilon_{fp} \delta, \\ \{\bar{\nabla}_{\dot{\alpha}}^f, \bar{\nabla}_{\dot{\beta}}^p\} &= -\epsilon_{\dot{\alpha}\dot{\beta}} \epsilon^{fp} \delta \end{aligned} \quad (2.8)$$

and have the following form

$$\begin{aligned}\nabla_f^\alpha &= \frac{\partial}{\partial \theta_f^\alpha} - i\partial^{\alpha\dot{\alpha}}\bar{\theta}_{\dot{\alpha}f} + \frac{1}{2}\theta_f^\alpha\delta, \\ \bar{\nabla}_{\dot{\alpha}}^f &= \frac{\partial}{\partial \theta_f^{\dot{\alpha}}} - i\partial_{\dot{\alpha}\alpha}\theta^{\alpha f} - \frac{1}{2}\bar{\theta}_{\dot{\alpha}}^f\delta.\end{aligned}\quad (2.9)$$

Here the action of central charge operator δ is trivial on Ψ , $\delta\Psi = 0$ and we can drop it in (2.6). We include it in (2.9) making preparations to the next section where it acts nontrivially on matter hypermultiplets. The invariant size of instanton is defined as

$$\rho_{inv}^2 = \rho^2 \left[1 + \frac{i}{\sqrt{2}} \Psi^a (\bar{\theta}_f \bar{u} \tau^a u \bar{\theta}^f) \right]. \quad (2.10)$$

Now let us comment on how different terms appear in (2.6) (see [11] for the detailed derivation). The first term in the exponent in (2.6) is the straightforward supersymmetrization of the first term in the exponent in (2.1). The second term is the supersymmetrization of Callan-Dashen -Gross term in (2.1). To see this, notice that two derivatives $\bar{\nabla}$ applied to $\bar{\Psi}^a$ contains $F_{\mu\nu}$ in component formulation. It is easy to see that this component coincides with the second term in the exponent in (2.1).

Now let us explain how the integration over full $N=2$ superspace arises in (2.6). It arises from the integration over fermion zero modes of instanton. In particular, instanton has eight $(4N_c + 2N_f)$ zero modes. Four zero modes come from one fermion field of the doublet λ^f and four another modes come from another one. In the $N=2$ supersymmetric Yang-Mills theory all Grassmann collective coordinates parametrizing fermion zero modes of instanton (normalized appropriately) have geometrical interpretation in terms of θ -parameters of superspace (compare with the $N=1$ case where not all Grassmann collective coordinates can be interpreted as θ -parameters [20, 21]). In particular, the two supersymmetric modes of $\lambda^1(\lambda^2)$ are proportional to $\theta^1(\theta^2)$, while two superconformal modes of $\lambda^1(\lambda^2)$ are proportional to $\bar{\theta}_2(\bar{\theta}_1)$.

Let us note that the reason for the name "superconformal zero modes" comes from $N=1$ SUSY gauge theories where these modes can be generated by superconformal transformation [20]. In $N=2$ SUSY this name is somewhat misleading. In fact, these modes can be viewed as supersymmetric. As we

mentioned above, they can be generated by the conjugate SUSY transformation [11].

The factor $1/\Psi^4$ appears in (2.6) as follows. After normalization of the measure in (2.6) in terms of superspace integral the factor $1/\langle \Psi \rangle^4$ appears. Then it is promoted to $1/\Psi^4$.

Note, that certain non-abelian effects which do not contribute at large distances are not included in (2.6).

Truncating the effective vertex (2.6) to include only light fields and integrating it over the size of instanton we get the one-instanton term of Seiberg-Witten solution for the prepotential

$$V_{YM}^F = \frac{\Lambda_0^4}{16\pi^2} \int d^4x d^4\theta \frac{1}{\mathcal{A}^2}, \quad (2.11)$$

as well as all powers of higher derivative corrections to (2.11)

$$V_{YM}^D = \frac{\Lambda_0^4}{8\pi^2} \int d^4x d^4\theta d^4\bar{\theta} \frac{d^3u}{2\pi^2 \mathcal{A}^4} \log \left[\bar{\mathcal{A}}\mathcal{A} + \frac{1}{4\sqrt{2}} (\bar{\nabla}_f \bar{u} \tau^3 u \bar{\nabla}^f) \bar{\mathcal{A}} \right] \quad (2.12)$$

where we use the notation $\mathcal{A} = \Psi_3$ for the light component of the gauge superfield.

Note, that the expansion parameter in (2.12) is ∇^2/\mathcal{A} , while uncontrollable corrections to (2.6) comes in powers of ρ^2/x^2 . These are down by the extra coupling constant $\rho \nabla^2 \sim g \nabla^2/\mathcal{A}$.

Let us note, in the conclusion of this section that the next to the leading order term in the momentum expansion of the effective action is given by the full superspace integral of a real function $K(\bar{\mathcal{A}}, \mathcal{A})$. The modular transformation properties of this function were studied in [22], while the perturbative contribution to this function was discussed in [23]. The one-instanton contribution to $K(\bar{\mathcal{A}}, \mathcal{A})$ [11] is given by (2.12) if we drop Callan-Dashen-Gross term in this equation. (Drop the second term in the argument of logarithm. It corresponds to higher corrections in the momentum expansion.) The proposal for the exact solution for $K(\bar{\mathcal{A}}, \mathcal{A})$ was suggested in [24].

Recently the function $K(\bar{\mathcal{A}}, \mathcal{A})$ has been studied in $N=2$ SUSY QCD with $N_f = 4$ hypermultiplets [25]. For $N_f = 4$ the theory is finite and $K(\bar{\mathcal{A}}, \mathcal{A})$ is given by its one loop contribution.

3 Instanton-induced vertex in the theory with matter

Now we include N_f matter hypermultiplets in our theory. In terms of $N=1$ superfields the matter dependent part of the microscopic action looks like

$$\begin{aligned} S_{matter} = & \frac{1}{g^2} \int d^4x d^2\theta d^2\bar{\theta} [\bar{Q}_A e^{-2V} Q^A + \bar{\tilde{Q}}^A e^{-2V} \tilde{Q}_A] \\ & + \frac{1}{g^2} \int d^4x d^2\theta [\sqrt{2}i \tilde{Q}_A \Phi Q^A + im \tilde{Q}_A Q^A] + \\ & + \frac{1}{g^2} \int d^4x d^2\bar{\theta} [\sqrt{2}i \bar{Q}_A \bar{\Phi} \bar{\tilde{Q}}^A + im \bar{Q}_A \bar{\tilde{Q}}^A], \end{aligned} \quad (3.1)$$

where Q^{kA} , \tilde{Q}_A^k are in the fundamental representation of the gauge group, $k=1, 2$, while $A=1, \dots, N_f$.

The instanton solution for the scalar component q^k of the matter superfield Q^k is given by [16, 26]

$$q_I^{kA} = \frac{\langle q^{kA} \rangle}{H^{1/2}}, \quad \tilde{q}_{IAk} = \frac{\langle \tilde{q}_{Ak} \rangle}{H^{1/2}}, \quad (3.2)$$

where we assume for a moment that the VEV $\langle q^{kA} \rangle \neq 0$ is developed.

In terms of instanton-induced effective vertex this field is generated by the insertion of the factor [15]

$$v_m = e^{-\frac{2\pi^2}{g^2}\rho^2[\bar{q}_{Ak}q^{kA} + \bar{\tilde{q}}_k^A \tilde{q}_A^k]} \quad (3.3)$$

in the integrand of our effective vertex (2.1) for boson fields. To see this we can calculate the correlation function

$$\langle q(x_1) \dots q(x_n) \rangle \quad (3.4)$$

in the instanton background. It is given by the product of classical solutions (3.2). Expanding (3.2) in the limit $(x_i - x_0)^2 \gg \rho^2$ it is easy to see that the product in (3.4) is reproduced by the vertex (3.3) in the framework of the perturbative theory.

Now let us supersymmetrize (3.3). We will go to it in two steps, first making the $N=1$ supersymmetrization and then presenting the $N=2$ supersymmetric vertex.

For the sake of simplicity we consider below the case $N_f = 1$. The generalization to the arbitrary N_f ($N_f \leq 3$) will be straightforward. The $N=1$ supersymmetric version of the vertex (3.3) looks like

$$v_m = \frac{g^2}{\pi^2} \int \frac{d^2 \bar{\theta}'_1}{\tilde{Q}Q} \frac{1}{\rho_{inv}^2} \exp \left\{ -\frac{2\pi^2}{g^2} \rho_{inv}^2 [\bar{Q}e^{-2V}Q + \tilde{\bar{Q}}e^{-2V}\tilde{Q}] \right\} \quad (3.5)$$

Each flavour of matter has two fermion zero modes. The Grassmann collective coordinates $\bar{\theta}'_1$, associated with these modes in appropriate normalization get shifted upon $N=1$ SUSY transformation [21] (this coordinate was first considered in [20])

$$\bar{\theta}'_1 \rightarrow \bar{\theta}'_1 - \bar{\epsilon}. \quad (3.6)$$

Hence it can be identified with $\bar{\theta}'_1$ coordinate of the superspace.

This is the reason for the appearance of the integral over $\bar{\theta}'_1$ in (3.5). Not to confuse this coordinate with $\bar{\theta}_1$ coordinate, which is already present in the effective vertex (2.6) for Yang-Mills theory, we put a prime on it. Fields Q , \tilde{Q} , in (3.5) are supposed to be functions of $x, \theta^1, \bar{\theta}'_1$.

The factor g^2 in the preexponent in (3.5) can be understood as follows. In general, the g^2 -dependence of the instanton measure is given by [27]

$$\left(\frac{1}{g^2} \right)^{\frac{n_b - n_f}{2}}, \quad (3.7)$$

where n_b and n_f is the number of boson and fermion zero modes of instanton. For the theory at hand $n_b = 8$, while $n_f = 8 + 2N_f$. This gives factor g^{2N_f} .

Other factors in the preexponent in (3.5) arise from the normalization of fermionic zero modes (see, for example [20]). In particular, $(\tilde{Q}Q)^{-1}$ arises in (3.5) because fermion zero modes ψ and $\tilde{\psi}$ proportional to VEV's $\langle Q \rangle$ and $\langle \tilde{Q} \rangle$. It arises in a similar way to the factor $1/\Psi^4$ in (2.6), which is associated with the integration over $\bar{\theta}_1$ and $\bar{\theta}_2$.

Now let us comment on the appearance of ρ_{inv}^2 in (3.5). One might think that as soon as we have two sets of coordinates $\bar{\theta}_1$ and $\bar{\theta}'_1$ we could construct two different versions of ρ_{inv}^2 using eq. (2.10). We will show later in this section that the only $N=2$ SUSY invariant expression for the size of instanton is given by (2.10).

Let us observe now that our vertex (3.5) is only $N=1$ supersymmetric. To make it $N=2$ supersymmetric we use $N=2$ superspace formalism (see, for

a review, [19]). We introduce $N=2$ hypermultiplet superfields Z^{kf} and \bar{Z}_{fk} which are doublets under the global $SU_R(2)$ group, $f = 1, 2$ ($k = 1, 2$ is a colour index). Lower components of its expansion in θ 's looks like

$$Z^f = q^f + \sqrt{2}\theta_\alpha^f \psi^\alpha + \sqrt{2}\bar{\theta}^{\dot{\alpha}f} \bar{\psi}_{\dot{\alpha}} + \dots \quad (3.8)$$

Here $q^1 = q$ and $q^2 = \bar{q}$. For the conjugated multiplet we have a similar expression

$$\bar{Z}_f = \bar{q}_f + \sqrt{2}\bar{\theta}_f^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} + \sqrt{2}\theta_{\alpha f} \tilde{\psi}^\alpha + \dots, \quad (3.9)$$

where $\bar{q}_1 = \bar{q}$ and $\bar{q}_2 = -\tilde{q}$.

Fields Z, \bar{Z} satisfy the following constraint

$$\begin{aligned} \nabla_f^\alpha Z^p &= \frac{1}{2}\delta_f^p \nabla_m^\alpha Z^m, \\ \bar{\nabla}_{\dot{\alpha}f} Z^p &= \frac{1}{2}\delta_f^p \bar{\nabla}_{\dot{\alpha}m} Z^m, \end{aligned} \quad (3.10)$$

which remove the isotriplet part of $\nabla^f Z^p$. In particular, the fermion fields in (3.8), (3.9) are $SU_R(2)$ -singlets. Superderivatives in (3.10) are given by (2.8), (2.9). The conjugate field \bar{Z} satisfy the similar constraint. The constraint (3.10) means that we are actually using on-shell superspace formulation. We will see some obstacles of this formalism later on in this paper.

In terms of $N=2$ superfields the obvious generalization of the exponential in (3.5) is

$$e^{-\frac{2\pi^2}{g^2} \rho_{inv}^2 \bar{Z}_{fk} Z^{kf}} \quad (3.11)$$

Let us check that the insertion of (3.11) in the instanton-induced vertex reproduces correctly fermion zero modes of instanton. To do so we calculate $\psi(x)$ in the instanton background, using (3.11). We have

$$\psi(x)_I = \langle \psi(x), e^{-\frac{2\pi^2}{g^2} \rho_{inv}^2 \bar{Z}_f Z^f(x_0)} \rangle. \quad (3.12)$$

The relevant term in the expansion of $\bar{Z}_f Z^f$ is

$$\bar{Z}_f Z^f \rightarrow \sqrt{2}\bar{\theta}_f^{\dot{\alpha}'} \bar{\psi}_{\dot{\alpha}}(x_0) \langle q^f \rangle \quad (3.13)$$

Here we introduce a new set of variables $\bar{\theta}'_f$ (in addition to $\bar{\theta}_f$ which are already present in (2.6)) related to matter fermion zero modes and assume

that fields Z, \bar{Z} depend on them. This is quite similar to what we have done in eq. (3.5) with the first supersymmetry $\bar{\theta}$ -parameter. Substituting (3.13) into (3.12) we find that up to gauge transformation ψ_I is given by the leading term of the expansion of

$$\psi_I^\alpha = i\sqrt{2}\nabla^{\alpha\dot{\alpha}}q_I\bar{\theta}'_{1\dot{\alpha}} + i\sqrt{2}\nabla^{\alpha\dot{\alpha}}\bar{q}_I\bar{\theta}'_{2\dot{\alpha}} \quad (3.14)$$

in powers of $\rho^2/(x-x_0)^2$. Here q_I is the instanton scalar field (3.2). Eq. (3.14) is the correct expression for the fermion zero mode of instanton. The reason is that we can get fermion zero mode ψ making first SUSY transformation of q_I or making the second SUSY transformation of \bar{q}_I . This corresponds to two terms in (3.14). We see that our $SU_R(2)$ -invariant vertex (3.11) reproduces the $N=2$ supersymmetric structure correctly. Note, that if we calculate ψ_I using the vertex (3.5) we would get only the first term in (3.14).

The relevant $SU_R(2)$ -invariant parameter which enters (3.13) or (3.14) is

$$w^{\dot{\alpha}k} = \bar{\theta}_f^{\dot{\alpha}'} < q^{fk} > = \bar{\theta}_1^{\dot{\alpha}'} < q^k > + \bar{\theta}_2^{\dot{\alpha}'} < \bar{q}^k >. \quad (3.15)$$

The conjugate parameter which emerges if we calculate $\bar{\psi}$ or $\tilde{\psi}$ looks like

$$\bar{w}_k^{\dot{\alpha}} = \bar{\theta}_f^{\dot{\alpha}'} < \bar{q}^f > = \bar{\theta}_1^{\dot{\alpha}'} < \bar{q}_k > + \bar{\theta}_2^{\dot{\alpha}'} < \bar{q}_k >. \quad (3.16)$$

We will see later that w, \bar{w} are the $SU_R(2)$ -invariant parameters which replace $\bar{\theta}'_1$ -parameters in the integration measure in (3.5).

Let us now come back to the issue of the SUSY invariant size of instanton. The SUSY transformation law for ρ is already fixed in $N=2$ Yang-Mills theory [11] and does not contain any $\bar{\theta}'_f$ -parameters associated with matter. Therefore, the only invariant combination which we can construct using ρ^2 is the one in (2.10). However, now we can construct SUSY invariant combination $(\bar{\theta}'_f - \bar{\theta}_f)$. With this taken into account the general form of the exponential which enters our effective vertex is

$$\exp\left\{-\frac{2\pi^2}{g^2}\rho_{inv}^2\bar{Z}_fZ^f[1+c\frac{i}{\sqrt{2}}\Psi^a((\bar{\theta}'_m - \bar{\theta}_m)\bar{u}\tau^a u(\bar{\theta}^{m'} - \bar{\theta}^m))]\right\} \quad (3.17)$$

where c is a constant.

We can restrict ourselves to the only quadratic in $(\bar{\theta}' - \bar{\theta})$ terms here because we have only two Grassmann parameters $\bar{\theta}'$ to integrate over in (3.5)

(later in this section we will work out the $N=2$ supersymmetric measure instead of the one in (3.5)).

Let us now fix the constant c . To do so it is sufficient to ignore quantum fluctuations of in (3.17) and replace all fields in (3.17) by their expectation values. Then the expression in the exponent in (3.17) is nothing other then the instanton action. In particular, to fix the coefficient c it is sufficient to consider terms of this action quadratic in $\bar{\theta}', \bar{\theta}$ parameters. These come from Yukawa couplings in (3.1). There are two Yukawa couplings in (3.1). The first type comes from kinetic terms and the second one comes from the superpotential. Substituting matter fermion zero modes (3.14) as well as λ^f ones (see, for example [11]) we get after some lengthy calculation

$$c = -1 \quad (3.18)$$

Now let us work out the $N=2$ supersymmetric integration measure of our effective vertex.

First of all we replace the integral over $\bar{\theta}'_1$ in (3.5) by the derivative over the same parameter

$$d\mu_m^{N=1} = \frac{d^2\bar{\theta}'_1}{<\tilde{q}><q>} \rightarrow -\frac{1}{4} \frac{\bar{\nabla}^{1\dot{\alpha}'} \bar{\nabla}^{1'}_{\dot{\alpha}}}{<\tilde{q}><q>} \quad (3.19)$$

where $\bar{\nabla}^f$ are given by (2.9). We assume here that derivatives act on variables $\bar{\theta}'$ in (3.17). Note that ρ_{inv}^2 is given by (2.10) and $\bar{\nabla}'$ do not act on it. After differentiation we put $\bar{\theta}'^f = \bar{\theta}^f$.

The reason for the substitution (3.19) is that acting with $\bar{\nabla}'$ is not equivalent any longer to the integration over $\bar{\theta}'$. In particular, these actions are different by terms with space derivatives (see (2.9)) which are no longer total derivatives because $\bar{\nabla}'$ acts on matter fields only.

As we pointed out above the matter fermion zero modes depend on Grassmann variables w, \bar{w} (3.15), (3.16). If we ignore the second supersymmetry these parameters reduce to $\bar{\theta}'_1 < q >, \bar{\theta}'_1 < \tilde{q} >$.

The $N=1$ measure (3.19) has the following property

$$\int d\mu_m^{N=1} \bar{\theta}'^{\dot{\alpha}}_1 < \tilde{q}_k > \bar{\theta}'_{1\dot{\alpha}} < q^k > = 1 \quad (3.20)$$

Now it is clear that the Grassmann parameters we have to integrate (or differentiate) in the matter dependent part of instanton measure are w and

\bar{w} . However, we would like to write down the instanton integration measure in terms of $\bar{\theta}'$ -parameters which has superspace interpretation. To do so we make the obvious $SU_R(2)$ -invariant generalization of the condition (3.20). Namely, $d\mu_m$ should satisfy the condition

$$\int d\mu_m \bar{w}_k^{\dot{\alpha}} w_{\dot{\alpha}}^k = 1 \quad (3.21)$$

The general form of the measure is fixed by $SU_R(2)$ symmetry up to two constants c_1 and c_2

$$d\mu_m = \frac{(<\bar{q}_f><q_p>)\frac{1}{2}(\bar{\nabla}^{f\dot{\alpha}}\bar{\nabla}_{\dot{\alpha}}^p + \bar{\nabla}^{p\dot{\alpha}}\bar{\nabla}_{\dot{\alpha}}^f)}{c_1 \det_{n,m}(<\bar{q}_n><q^m>) + c_2(<\bar{q}_n><q^n>)^2}, \quad (3.22)$$

where we assume the contraction of colour induces inside the brackets.

We write down the symmetric in f and p combination of $\bar{\nabla}^f\bar{\nabla}^p$ here. The antisymmetric one reduces to the action of the central charge δ (see (2.8)). We postpone the discussion of the possibility to add the central charge operator to the instanton measure (3.22) till the next section.

To fix constants c_1 and c_2 in (3.22) we substitute (3.22) in (3.21). This gives

$$d\mu_m = -\frac{1}{2} \frac{(<\bar{q}_f><q_p>)\frac{1}{2}(\bar{\nabla}^f\bar{\nabla}^p + \bar{\nabla}^p\bar{\nabla}^f)}{4\det_{n,m}(<\bar{q}_n><q^m>) - (<\bar{q}_m><q^m>)^2}, \quad (3.23)$$

Now putting together the instanton vertex (3.17), the integration measure (3.23) and combining it with the vertex in (2.6) we arrive at

$$\begin{aligned} V_{N_f=1} = & -\frac{\Lambda_1^3}{4\pi^2} \int d^4x \frac{d\rho}{\rho} \frac{d^3u}{2\pi^2} d^4\theta d^4\bar{\theta} \frac{1}{\Psi^{a4}} \\ & \exp\left[-\frac{4\pi^2}{g^2} \rho_{inv}^2 \bar{\Psi}^a \Psi^a - \frac{\pi^2}{\sqrt{2}g^2} \rho_{inv}^2 i(\bar{\nabla}_f \bar{u} \tau^a u \bar{\nabla}^f) \bar{\Psi}^a\right] \\ & \left(-\frac{g^2}{2\pi^2 \rho_{inv}^2}\right) \frac{(\bar{Z}_f Z_p)}{4\det_{n,m}(\bar{Z}_n Z^m) - (\bar{Z}_m Z^m)^2} \frac{1}{2} \{\bar{D}'^f, \bar{D}'^p\} \\ & \exp\left[-\frac{4\pi^2}{g^2} \rho_{inv}^2 (\bar{Z}_n Z^n)\right] |_{\bar{\theta}'=\bar{\theta}}, \end{aligned} \quad (3.24)$$

where we also replace $\bar{\nabla}_{\dot{\alpha}}^f$ by covariant superderivative $\bar{D}_{\dot{\alpha}}^f = (\bar{\nabla}_{\dot{\alpha}}^f + V_{\dot{\alpha}}^f)$ [17].

This is our final result for the instanton-induced effective vertex in the theory with $N_f = 1$ flavours. The scale of this theory Λ_1 appears in (3.24) in the power $4 - N_f = 3$. We drop in (3.24) the term which contains explicit dependence on $\bar{\theta}' - \bar{\theta}$, see (3.17). The reason for that is that under the action of $\{\bar{\nabla}^{f\dot{\alpha}}, \bar{\nabla}_{\dot{\alpha}}^p\}$ this term is proportional to $Tr(\bar{u}\tau^b u) = 0$. We also promote expectation values of fields to their actual values in the integration measure in (3.24).

So far we assumed that matter fields develop their expectation values. However, this can be considered just as a technical trick to derive (3.24). Now we relax this condition. In the next section we use effective vertex (3.24) in different regions of the modular space, in particular on the Coulomb branch where $\langle Z \rangle = 0$. Note moreover, that we considered the theory with $N_f = 1$ in this section. Strictly speaking, in this case Higgs branches are not developed at all.

Promoting matter VEV's to Z-fields in the preexponential in (3.24) rises the following problem. Can derivatives \bar{D}' in (3.24) act on fields Z, \bar{Z} in the preexponent as well as on the exponential? We cannot answer this question in the semiclassical approximation. It requires a study of a next-to-leading order effects in g^2 . However, we will argue in the next section that we get reasonable $N=2$ supersymmetric results if \bar{D}' 's act on the exponential only as it is written down in (3.24).

4 Low energy effective action

In this section we use the instanton-induced vertex (3.24) to work out one-instanton terms in the low energy effective Lagrangian for light fields.

First of all let us consider the region $\langle A \rangle \gg \Lambda_{N_f}$ on the Coulomb branch in the limit m goes to infinity $m \gg \langle A \rangle$. Matter fields in this limit can be integrated out in (3.24) and we should recover the vertex (2.6) for $N_f = 0$ theory.

To integrate out matter fields we consider the correlation function of the mass term with the vertex (3.24) and afterwards put matter fields to zero

$$V_{N_f=0} = \langle -\frac{1}{g^2} \int d^4x d^2\theta m \tilde{Q} Q, V_{N_f=1} \rangle |_{Z=\bar{Z}=0}. \quad (4.1)$$

Note, that the conjugate mass term $\bar{m} \tilde{Q} \bar{Q}$ do not contribute (it gives nonzero

correlation function with the anti-instanton vertex which is conjugate to (3.24)). The only nonzero contribution to (4.1) comes from the fermion loop. The operator $\{\bar{\nabla}^f, \bar{\nabla}^p\}$ acting on the exponential in (3.24) reduces in the leading order in g^2 to

$$\begin{aligned} \bar{\nabla}^f \bar{\nabla}^p e^{-\frac{2\pi^2}{g^2} \rho_{inv}^2 \bar{Z}Z} &\rightarrow -\frac{8\pi^4}{g^4} \rho_{inv}^4 \left[(\bar{Z}^f \bar{\psi}^{\dot{\alpha}})(\bar{\psi}_{\dot{\alpha}} Z^p) \right. \\ &\quad \left. + (\bar{Z}^p \bar{\psi}^{\dot{\alpha}})(\bar{\psi}_{\dot{\alpha}} Z^f) \right], \end{aligned} \quad (4.2)$$

while the mass term in (4.1) reduces to $m\tilde{\psi}\psi$. Contracting fermion fields in the mass term with fermion fields in the expansion (4.2) of $V_{N_f=1}$ we see that the structure of matter fields which appears in the numerator is

$$4\det(\bar{Z}Z) - (\bar{Z}Z)^2. \quad (4.3)$$

This is cancelled against the same factor in the denominator of the instanton measure in (3.24). Now we can safely put $\bar{Z} = Z = 0$. The only remaining problem is that the fermion loop integral appears to be *UV* divergent:

$$\langle m\tilde{\psi}\psi, V_{N_f=1} \rangle \sim m \int d^4x_0 \frac{1}{(x-x_0)^6}. \quad (4.4)$$

This means that this integral is dominated at short distances $(x-x_0)^2 \sim \rho^2$ and strictly speaking we cannot use our effective vertex (3.24) to calculate correlation function (4.1). However this problem can be easily resolved. Effective vertex (3.24) gives us only the leading term in the expansion of fermion zero mode (3.14) in powers of $\rho^2/(x-x_0)^2$. Replacing the approximate expressions of $\psi, \tilde{\psi}$ in (4.4) by the exact ones using (3.14) we get

$$m \int d^4x_0 \frac{1}{(x-x_0)^6} \rightarrow m \int d^4x_0 \frac{1}{(x-x_0)^6 H^3(x-x_0)}. \quad (4.5)$$

This makes the integral convergent. Putting all factors together we see that (4.1) reduces to the instanton-induced vertex for the Yang-Mills theory (2.6) with the scale

$$\Lambda_0^4 = m\Lambda_1^3. \quad (4.6)$$

This is a correct result in the Pauli-Villars regularization scheme we use in this paper [7]. We consider the calculation above as a non-trivial check on our effective vertex (3.24).

Let us now consider the region on the Coulomb branch close to the charge singularity, $\langle A \rangle \rightarrow -\sqrt{2}m$. We keep $m \gg \Lambda_{N_f}$ to ensure the weak coupling regime. The light fields in this region are: $U(1)$ gauge multiplet as well as a charged matter hypermultiplet. We can decompose matter field as

$$Z^{kf} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^k Z_+^f + \begin{pmatrix} 0 \\ 1 \end{pmatrix}^k Z_-^f. \quad (4.7)$$

Near the singularity field Z_+ becoming light. Its mass goes like

$$m' = m + \frac{1}{\sqrt{2}} \langle \mathcal{A} \rangle \rightarrow 0. \quad (4.8)$$

Field Z_- remains massive. Now we truncate our effective vertex (3.24) replacing Ψ^a by $\mathcal{A} = \Psi^3$ and Z^{kf} by Z_+^k . Note, that this simple recipe is correct only in the leading order in g^2 . In higher orders in g^2 loop graphs with massive particles propagating in loops have to be taken into account.

Note also, that, in principle, we can integrate out massive fields Z_- at the one loop level using their mass term insertion like we have done it above for matter multiplet far away from the charge singularity. However, it is easy to show that this contribution is zero.

Let us now suppress the subscript (+) of the matter field and put $Z_+^f = Z^f$, assuming from now on that Z^f carries no colour index.

Now to get the low energy effective Lagrangian from (3.24) let us integrate over the instanton size ρ . To do so let us act with $\{\bar{D}^f, \bar{D}^p\}$ on the exponential of matter fields in (3.24). Two different structures of matter fields appears. The first one is

$$\frac{\bar{Z}_f \delta Z^f - \delta \bar{Z}_f Z^f}{\bar{Z} Z} e^{-\frac{2\pi^2}{g^2} \rho_{inv}^2 \bar{Z} Z}, \quad (4.9)$$

while the second one looks like

$$\frac{\rho_{inv}^2}{g^2} (\bar{D}^n \bar{Z}_n) (\bar{D}^m Z_m) e^{-\frac{2\pi^2}{g^2} \rho_{inv}^2 \bar{Z} Z}. \quad (4.10)$$

To get (4.9), (4.10) we used the constraint (3.10). In particular, (4.9) arise when two derivatives act on the same field. Using the identities

$$\bar{\nabla}^f \bar{\nabla}^p Z_n = -2\delta_n^p \delta Z^f, \quad \bar{\nabla}^f \bar{\nabla}^p \bar{Z}_n = -2\delta_n^p \delta \bar{Z}^f, \quad (4.11)$$

this contribution can be reduced to the action of the central charge (see (4.9)). Contribution (4.10) arises when two derivatives act on different fields.

Now observe that (4.9) goes to zero on the mass shell. To see this recall that equations of motion for matter fields read

$$\delta Z^f = 2im'Z^f, \quad \delta \bar{Z}^f = -2im'\bar{Z}^f, \quad (4.12)$$

where the mass of light multiplet (4.8) goes to zero. This means that on the mass shell (i.e., if equations of motion (4.12) are fulfilled) we can ignore the contribution (4.9) as compared with (4.10). Strictly speaking, we cannot use equations of motion in quantum theory. However, terms which are zero on equations of motion produce δ -functional contributions to correlation functions. They cannot be seen in the large distance limit we are working in in the low energy effective theory. Therefore in what follows we ignore contribution (4.9).

Note, moreover, that the integral over ρ^2 in (3.24) associated with (4.9) contains a logarithmic divergent piece in UV . If we take (4.9) seriously, this would be a new UV divergence which emerges at the non-perturbative level. We believe that there are no such divergences in four dimensional gauge theories.

Now let us integrate over ρ^2 in (3.24) keeping only the long-range contribution (4.10). To do so, note, that the integral over $\bar{\theta}$ -parameters in (3.24) can be saturated either by the explicit dependence of ρ_{inv}^2 on $\bar{\theta}$'s in (2.10) or by the $\bar{\theta}$ -dependence of fields \bar{A} , \bar{Z} and Z . The first contribution can be analysed as follows [11]. For any function f we have

$$\int d^4\bar{\theta} f(\rho_{inv}^2) = -\frac{1}{2}\mathcal{A}^2 \left[\left(\rho^2 \frac{\partial}{\partial \rho^2} \right)^2 - \rho^2 \frac{\partial}{\partial \rho^2} \right] f(\rho^2) \quad (4.13)$$

Eq. (4.11) shows that the integral over ρ^2 reduces to total derivative. Now eq. (4.10) shows that contributions from both limits $\rho^2 \rightarrow 0$ and $\rho^2 \rightarrow \infty$ is zero. This is in contrast with the case of pure Yang-Mills theory where the Seiberg-Witten contribution (2.11) comes from zero size instanton [11].

Now we ignore $\bar{\theta}$ -dependence of ρ_{inv}^2 putting $\rho_{inv}^2 \rightarrow \rho^2$. Integrating over ρ^2 we get

$$V_{N_f=1}^{Root} = -\frac{1}{4} \frac{\Lambda_1^3}{8\pi^2} \int d^4x d^4\theta d^4\bar{\theta} \frac{d^3u}{2\pi^2} \frac{1}{\mathcal{A}^4}$$

$$\times \frac{(\bar{D}^n \bar{Z}_n)(\bar{D}^m Z_m)}{2\bar{\mathcal{A}}\mathcal{A} + \bar{Z}_f Z^f + \frac{i}{2\sqrt{2}}(\bar{\nabla}_p \bar{u} \tau^3 u \bar{\nabla}^p) \bar{\mathcal{A}}}. \quad (4.14)$$

Let us address the following question. Is our result in (4.14) $N=2$ supersymmetric? The answer is that it is supersymmetric only on-shell. In general, the full superspace integral of some $N=2$ superfield is superinvariant if the central charge of this superfield is zero. It is easy to see that the operator δ acting on the integrand in (4.14) gives zero only if we use equations of motions (4.12). This is the main drawback of the on-shell superspace formalism we use in this paper. Presumably, completely supersymmetric result for the instanton-induced Lagrangian can be obtained in the harmonic superspace formulation [28].

However, as we explained above we can ignore terms which are zero on equations of motion in the low energy effective Lagrangian and consider (4.14) as being $N=2$ supersymmetric.

Another problem we would like to discuss now related to the possibility that spinor derivatives in (3.24) could, in principle, act on (some of) matter fields in the preexponent as well as on the exponential.

This would produce a disastrous consequences in our theory. First of all the integral over ρ^2 would give us the UV logarithmic divergence associated now with long-range term

$$\frac{(\bar{D}^n \bar{Z}_n)(\bar{D}^m Z_m)}{\bar{Z}Z} \log a^2, \quad (4.15)$$

where $1/a$ is the UV cutoff. Second, this would produce a non-zero contribution coming from $\bar{\theta}$ -dependence of ρ_{inv}^2 . Using eq. (4.13) we would get a long-range term of the type

$$\int d^4x d^4\theta \frac{1}{\mathcal{A}^2} \frac{(\bar{D}^n \bar{Z}_n)(\bar{D}^m Z_m)}{\bar{Z}Z} \quad (4.16)$$

The contribution (4.16) would break the supersymmetry explicitly because it is a half superspace integral of a field which is not a chiral superfield.

As we have explained in the previous section, we cannot answer the question whether $\bar{\nabla}$'s in (3.24) should act on matter fields in the preexponent or not in the semiclassical approximation. However, we can get rid of both unwanted contributions (4.15) and (4.16) if we say that $\bar{\nabla}$'s act on the exponential of matter fields only as it stands in (3.24). That is what we are going to do in what follows.

Now let us generalize our result (4.14) for $N_f = 1$ theory to the case of arbitrary $N_f \leq 3$. Substituting the product of factors (4.10) instead of matter dependent factor in (3.24) we get

$$V_{N_f}^{Root} = -\frac{(N_f - 1)!}{4^{N_f}} \frac{\Lambda_{N_f}^{4-N_f}}{8\pi^2} \int d^4x d^4\theta d^4\bar{\theta} \frac{d^3u}{2\pi^2} \frac{1}{\mathcal{A}^4} \frac{\det_{A,B}[(\bar{D}^n \bar{Z}_{An})(\bar{D}^m Z_m^B)]}{[2\bar{\mathcal{A}}\mathcal{A} + \bar{Z}_{Cf} Z^{fC} + \frac{i}{2\sqrt{2}}(\bar{\nabla}_p \bar{u} \tau^3 u \bar{\nabla}^p) \bar{\mathcal{A}}]^{N_f}}, \quad (4.17)$$

where $A, B, C = 1, \dots, N_f$ are flavour indices. This is our final result for the one-instanton contribution to the low effective Lagrangian on the Coulomb branch near the root of the charge singularity.

Like the one in (2.12) for the pure Yang-Mills theory the Lagrangian (4.17) contains all powers of derivatives of the gauge field $\bar{\mathcal{A}}$. However, there is no expansion in derivatives of matter fields in (4.17). Note that, in principle, we control all possible powers of derivatives of matter fields as well as derivatives of the gauge field in our approximation.

Another comment related to the result (4.17) is that, as we explained above, the VEV's of matter fields are zero on the Coulomb branch. The effective Lagrangian (4.17) is not singular because the VEV of the gauge multiplet is not zero $\langle \mathcal{A} \rangle \rightarrow -\sqrt{2}m$. In particular, the singularity at $Z \rightarrow 0$ which is present in the instanton measure in (3.24) is cancelled out in (4.17).

Now let us consider the low energy effective Lagrangian on the Higgs branch which emerges from the charge singularity. The $SU(2)$ gauge symmetry is completely broken on the Higgs branch and gauge particles are massive. In particular, the photon multiplet acquires a mass and should not be included any longer in the low energy effective theory. Thus we put

$$\mathcal{A} = -\sqrt{2}m. \quad (4.18)$$

The conditions on possible VEV's of matter fields come from putting D-terms and F-terms to zero [3]. In $SU_R(2)$ -invariant form these conditions look like

$$\bar{Z}_{Ap}(\tau^a)_f^p Z^{fA} = 0, \quad (4.19)$$

where $a = 1, 2, 3$. These equations have nonzero solutions for $N_f \geq 2$. In what follows we consider the case $2 \leq N_f < 4$. Let us consider also the

region of large VEV of matter field

$$\langle \bar{Z}_{Af} Z^{fA} \rangle \gg |m|^2, \quad (4.20)$$

far away from singularity at $Z=0$. Substituting (4.18), (4.20) in (4.17) we get the effective Lagrangian on the Higgs branch

$$V_{N_f}^{Higgs} = -\frac{(N_f - 1)!}{4^{N_f}} \frac{\Lambda^{4-N_f}}{32\pi^2 m^4} \int d^4x d^4\theta d^4\bar{\theta} \frac{\det_{A,B}[(\bar{\nabla}^{\dot{\alpha}n} \bar{Z}_{An})(\bar{\nabla}^m_{\dot{\alpha}} Z_m^B)]}{(\bar{Z}_{Cf} Z^{fC})^{N_f}}, \quad (4.21)$$

where we replace covariant spinor derivatives with ordinary ones (2.9). We see that instanton induces a single term in the momentum expansion of the effective Lagrangian. In fact, because of the constraint (3.10), $SU_R(2)$ indices of $\bar{\nabla}$'s are always contracted with indices of matter fields. Therefore, by Pauli principle the number of $\bar{\nabla}^n Z_n$'s cannot be more than $2N_f$ and the number of $\bar{\nabla}^n \bar{Z}_n$'s also cannot be more than $2N_f$. If we consider the integral over $\bar{\theta}$ space as another four derivatives then the number of $\bar{\nabla}^n Z_n$'s in (4.21) is $2 + N_f$ and the number of $\bar{\nabla}^n \bar{Z}_n$'s is $2 + N_f$. We see that (4.21) is zero for $N_f = 1$ by Pauli principle. This is consistent with the fact that we have no Higgs branch for $N_f = 1$ at all. For $N_f = 2$ the number of fermion operators equals to the maximum possible number and we cannot have more $\bar{\nabla}$'s then it appears in (4.21).

The result (4.21) means that although the Higgs branch is a hyper-Kahler manifold and its metric has neither perturbative nor instanton corrections [3] it does receive a higher derivative correction (4.21). In components (4.21) produces $8 + 2N_f$ fermion terms (or, say, $2N_f$ fermions plus four space-time derivative terms).

5 Exact solution on the Higgs branch

In the previous section we worked out the one- instanton induced contribution (4.21) to the effective Lagrangian for light fields on the Higgs branch. Now we are going to consider result (4.21) as an asymptotic expression for a certain operator on the Higgs branch in the weak coupling limit of large mass $|m| \gg \Lambda_{N_f}$. In this section we work out the exact solution for this operator

at arbitrary complex values of m including the strong coupling region at $|m| \sim \Lambda_{N_f}$.

First of all let us write down the general form of the operator under consideration for arbitrary m . We still assume that we are working on the Higgs branch far away from the singularity at $Z = 0$, thus the conditions (4.18) and

$$\langle \bar{Z}_{Af} Z^{fA} \rangle \gg \Lambda_{N_f}^2 \quad (5.1)$$

are fulfilled. To write down the general form of the operator which gives (4.21) in the limit $m \rightarrow \infty$ compatible with symmetries of the theory let us work out $U_R(1)$ charges of different fields which appear in (4.21). Under $U_R(1)$ transformation $N=2$ superfields transforms as [2]

$$\begin{aligned} \mathcal{A} &\rightarrow e^{2i\alpha} \mathcal{A}, \quad \theta \rightarrow e^{i\alpha} \theta \\ \bar{\mathcal{A}} &\rightarrow e^{-2i\alpha} \bar{\mathcal{A}}, \quad \bar{\theta} \rightarrow e^{-i\alpha} \bar{\theta} \\ Z &\rightarrow Z, \quad \bar{Z} \rightarrow \bar{Z}. \end{aligned} \quad (5.2)$$

Mass term breaks $U_R(1)$ symmetry. However, we can think that it is conserved if we promote mass of matter to the background field [29] with the transformation law

$$m \rightarrow e^{2i\alpha} m \quad (5.3)$$

In a similar way anomaly ensures that instanton breaks $U_R(1)$ symmetry by $8 - 2N_f$ units. We can think that it is conserved if we promote the scale parameter Λ_{N_f} to the background field with the transformation law [30]

$$\Lambda_{N_f} \rightarrow e^{2i\alpha} \Lambda_{N_f}. \quad (5.4)$$

Observe now that the net $U_R(1)$ charge of the Lagrangian (4.21) is zero. The charge of $\bar{\nabla}$'s is $2N_f$. Combined with the charge of $\Lambda_{N_f}^{4-N_f}$, it gives 8 which is combined to zero with the charge of the mass factor m^{-4} .

Taking into account the $U_R(1)$ symmetry we now can write down the general form of the operator in question for arbitrary m . It has the form

$$V_{N_f}^{Higgs} = Y(m) \int d^4x d^4\theta d^4\bar{\theta} \frac{\det_{A,B}[(\bar{\nabla}^n \bar{Z}_{An})(\bar{\nabla}^m Z_m^B)]}{(\bar{Z}_{Cf} Z^{fC})^{N_f}}. \quad (5.5)$$

Here the function Y looks like

$$Y(m) = \frac{1}{m^{N_f}} J(\Lambda_{N_f}/m), \quad (5.6)$$

where J is the function of the ratio Λ_{N_f}/m . In particular, multi-instanton contributions to the function $Y(m)$ are given by (1.4), where the k -th term comes from the instanton with topological charge k . From this point of view our result (4.21) is the result for the coefficient J_1 in the expansion (1.4). Namely, (4.21) gives

$$J_1 = -\frac{(N_f - 1)!}{4^{N_f}} \frac{1}{32\pi^2} \quad (5.7)$$

Note, that (5.5) is not the only possible operator on the Higgs branch. We study the operator (5.5) in this paper because our instanton calculation shows that it is nonzero.

Now we are going to argue that the function $Y(m)$ is an analytic function of m , i.e. it does not depend on \bar{m} . Essentially, this means that $Y(m)$ is given by the instanton expansion (1.4) and do not receive perturbative or instanton-anti-instanton corrections.

Consider first possible higher loops perturbative corrections to instanton contributions in (1.4). They come proportional to the gauge coupling constant $g^2 \sim (\log \bar{Z}Z/\Lambda_{N_f}^2)^{-1}$ which is small, $g^2 \ll 1$, in the region of large VEV's of matter (5.1). Note, that coupling constant appears to be function of $\bar{Z}Z$, rather then $|m|^2$ because all massive particles acquire mass of order $\langle \bar{Z}Z \rangle$ on the Higgs branch, $\langle \bar{Z}Z \rangle \gg |m|^2$. We are in a wonderland where instanton corrections are more important then higher loops of perturbative theory.

What about one-loop contribution? To put it in another way do we have term with $k = 0$ in the expansion (1.4). The answer is no and the reason is that we have no particles with mass of order m which could produce the behaviour $Y(m) \sim 1/m^{N_f}$ in perturbative theory. We conclude that

$$J_0 = 0 \quad (5.8)$$

in expansion (1.4).

Now let us comment on possible instanton-anti-instanton corrections. These would produce powers of Λ_{N_f}/\bar{m} which would spoil the analyticity of the function $Y(m)$. Of course, it is always hard to study instanton-anti-instanton effects. However, here we are going to argue that they do not contribute to $Y(m)$.

Suppose we add an instanton-anti-instanton pair to the one-instanton calculation we have considered in the previous section. It can be studied within

the perturbative theory using the effective vertex (3.24) which describes an instanton together with the conjugate to (3.24) vertex which describes an anti-instanton [15, 21]. The typical Feynman graph involves extra powers of gauge coupling (coming from propagation functions in our normalization) which is small in the region (5.1) on the Higgs branch. The only exception to this arises if the integral over instanton-anti-instanton separation is divergent at small separations. Since the cutoff parameter for the effective vertex (3.24) is the size of instanton $\rho^2 \sim g^2 / \langle \bar{Z}Z \rangle$, this could result in the cancellation of powers of g^2 coming from propagation functions and could produce the unwanted effect of $0(1)$. This is exactly what happens if one tries to calculate, say, the two-instanton contribution to the prepotential in pure $N=2$ Yang-Mills theory using two one-instanton vertices (2.6). The result in this case comes from small separations between two instantons ($\sim \rho$) [8]. However, this cannot happen for instanton-anti-instanton pair. The reason is that instanton-anti-instanton pair becomes a trivial configuration at small separations and cannot produce such effect [31].

Now assuming that $Y(m)$ is analytic function of m and imposing its behaviour at large m , coming from one-instanton calculation

$$Y(m) = J_1 \frac{\Lambda_{N_f}^{4-N_f}}{m^4} + O\left(\frac{1}{m^{8-N_f}}\right) \quad (5.9)$$

with J_1 given by (5.7), we can find the exact solution for $Y(m)$ at arbitrary m .

To do so we have to assume certain singular behaviour of $Y(m)$ in the strong coupling region of $|m| \sim \Lambda_{N_f}$. The general idea is the following. Singularities of each term in the momentum expansion of the effective Lagrangian come from certain particles becoming massless. Generally speaking, those particles which produce logarithmic singularities to prepotential produce also power singularities to higher derivative operators. In fact, logarithmic singularities in the prepotential come from loop graphs with two external legs and light particle going around the loop. Power singularities in higher derivative operators come from loop graphs with more than two external legs and the same light particle going around the loop. The first natural conjecture to start with is that there are no other singularities in higher derivative terms.

In other words we conjecture that singularities of higher derivative operators come only from monopole, dyon or charge becoming massless. In

particular, we show below that singularities of $Y(m)$ arise when the root of Higgs branch collide with points on the Coulomb branch where monopole or dyon become massless. These colliding singularities were studied in [14]). On the Coulomb branch they correspond to conformal fields theories which describes mutually non-local particles becoming massless. Consider, first, values of m at which charge singularity collides with monopole (dyon) singularity. We consider the case $N_f = 2$ from now on. In terms of variable $u = \frac{1}{2}\Phi^{a^2}$ the position of charge singularity is given by

$$u_0 = m^2 + \frac{1}{2}\Lambda_2^2. \quad (5.10)$$

Here we use Pauli-Villars regularization scheme which define the scale Λ_{N_f} different from those used in [2, 3] (see [7] for a relation between different scales). The monopole (dyon) singularity is located at

$$u_{1,2} = \pm 2m\Lambda_2 - \frac{1}{2}\Lambda_2^2. \quad (5.11)$$

The charge singularity collides with monopole (dyon) one when $u_0 = u_{1,2}$. Substituting here (5.10) and (5.11) we find a quadratic equation which l.h.s. is a perfect square

$$(m \pm \Lambda_2)^2 = 0. \quad (5.12)$$

Hence, we have two two-fold degenerative solutions

$$m = \pm \Lambda_2. \quad (5.13)$$

We will see below that the two-fold degeneracy of solutions to (5.12) play an important role in fitting together strong coupling singularities of $Y(m)$ with its behaviour at large m .

Now recall that we are interested in finding the operator (5.5) at the Higgs branch, thus $u = u_0$, where u_0 is given by the equation (5.10). When m is close to colliding values of mass (5.13) an extra monopole (dyon) becomes light. Its mass near point (5.13) is given by the mass formula for the BPS states. Say, for monopole we have $\mu = \sqrt{2}a_D$. Now, near the monopole singularity the dual potential is [2, 3]

$$a_D \approx \frac{c_0(m)}{\Lambda_2}(u - u_1) = \frac{c_0(m)}{\Lambda_2}(m - \Lambda_2)^2, \quad (5.14)$$

where we put $u = u_0$. Here $c_0(m)$ is a function of m which can be extracted from Seiberg-Witten solution for the prepotential [3]. In fact, this function is singular at $m \rightarrow \Lambda_2$. The reason is that the anomalous dimension of a_D is one near conformal point, whereas the anomalous dimension of $m - \Lambda_2$ is $2/3$ [14]. This means that function $c_0(m)$ behaves like $c_0(m) \sim (m - \Lambda_2)^{-1/2}$ at $m \rightarrow \Lambda_2$. We get for the monopole (dyon) mass near point (5.13)

$$\mu \approx \frac{c_0}{\Lambda_2^{1/2}} (m \mp \Lambda_2)^{3/2}, \quad (5.15)$$

where c_0 is a calculable constant. Note, also that monopole (dyon) do not acquire a large mass $\sim <\bar{Z}Z>$ on the Higgs branch because of F -term conditions (see (4.19)).

Now let us estimate the behaviour of $Y(m)$ near the point of colliding singularities (5.13). Let us focus, say, on the fermion component of operator (5.5). We have overall eight $\bar{\nabla}$'s and four ∇ 's for $N_f = 2$ in (5.5). This gives the following fermion structure

$$V_{N_f=2}^{Higgs} \sim Y(m) \int d^4x \frac{\bar{\psi}^8 \psi^4}{<\bar{q}q>^6}, \quad (5.16)$$

where ψ is a symbolic notation for $\psi^A, \tilde{\psi}^A$.

Let us reproduce the operator (5.16) in the effective theory near the colliding point (5.13). Our effective theory near this value of m on the Higgs branch is $N = 2, N_f = 2$ QED with extra light monopole (dyon) hypermultiplet with mass (5.15). Unfortunately, there is no systematic way to treat theories with mutually non-local degrees of freedom. However, there are different descriptions of these theories in the Abelian case [32]. They based on the introduction of a space-time vector n_μ which essentially represents a Dirac string of a monopole. Although the Lorentz invariance is broken by this vector at any intermediate stage of the calculation, it can be shown that the physical observables do not depend on n_μ , provided the Dirac quantization condition is fulfilled [32].

What we need here from this theory is the existence of four-fermion vertex of type

$$\frac{1}{<\bar{q}q>} \bar{\psi}_A \psi^A \bar{\chi} \chi, \quad (5.17)$$

which can be thought as mediated by massive gauge boson exchange (with mass $\langle \bar{q}q \rangle$), as well as a four-fermion interaction

$$\frac{1}{\langle \bar{q}q \rangle} \bar{\psi}_A \tilde{\psi}^A \tilde{\chi} \chi, \quad (5.18)$$

which is mediated by massive adjoint scalar exchange (due to Yukawa couplings). Here χ denotes the fermion component of monopole (dyon) hypermultiplet.

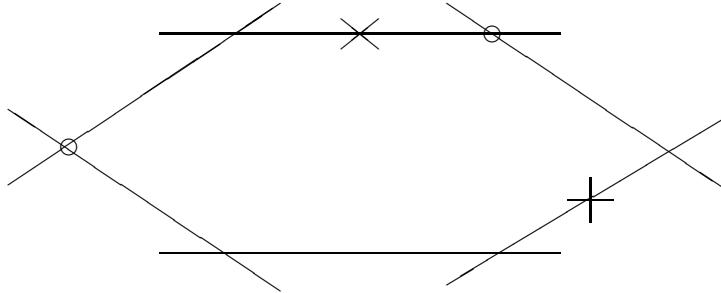


Fig. 1. Circles denote insertions of vertex (5.18),
while crosses denote mass insertions.

Now consider the one-loop graph with two vertices (5.18) and four vertices (5.17), see Fig. 1. External legs correspond to the fermion components of charge massless hypermultiplet ψ , while the fermion component of the

monopole field χ propagates around the loop. This loop graph gives

$$\frac{\bar{\psi}^8 \psi^4}{<\bar{q}q>^6} \int d^4k \frac{k^4 \bar{\mu}^2}{(k^2 + |\mu|^2)^6}, \quad (5.19)$$

where k is the momentum of the monopole. We need at least two $\bar{\mu}$ insertions in (5.19) to balance the chiral charge of monopole fermions induced by vertices (5.18). Performing the integral over momentum in (5.19) we get an estimate

$$\frac{1}{\mu^2} \frac{\bar{\psi}^8 \psi^4}{<\bar{q}q>^6}. \quad (5.20)$$

comparing (5.20) with (5.16) we get the behaviour of $Y(m)$ at $\mu \rightarrow 0$

$$Y(m) \sim \frac{c_{\pm}}{\mu^2}. \quad (5.21)$$

Here c_{\pm} are unknown constants for the monopole and the dyon case. To fix them we need a somewhat more detailed description of the effective theory of mutually non-local light particles. This goes beyond the scope of this paper.

Substituting (5.15) into (5.21) we get the behaviour of $Y(m)$ near points of colliding singularities (5.13)

$$Y(m) \sim \frac{c_{\pm} \Lambda_2}{c_0^2 (m \mp \Lambda_2)^3}, \quad m \rightarrow \pm \Lambda_2 \quad (5.22)$$

Now let us find the exact solution for $Y(m)$. Note, that we assume that singularities (5.22) at $m \rightarrow \pm \Lambda_2$ are the only singularities of $Y(m)$ at strong coupling. Then the obvious suggestion for the exact solution is

$$Y(m) = \frac{c \Lambda_2}{c_0^2 (m - \Lambda_2)^3} - \frac{c \Lambda_2}{c_0^2 (m + \Lambda_2)^3}. \quad (5.23)$$

Here we put $c_+ = -c_- = c$. The reason is that function $Y(m)$ is even, $Y(-m) = Y(m)$. This follows from eq.(1.4) which shows that the expansion of $Y(m)$ goes in even powers of m , at $N_f = 2$.

Observe now, that the solution in (5.23) reproduces the behaviour (5.9) of $Y(m)$ at large m coming from the instanton calculation. Note, that if there were no two-fold degeneracy of solutions to (5.12) the function $Y(m)$ would behave like $O((m \mp \Lambda_2)^{-3/2})$ instead of $O((m \mp \Lambda_2)^3)$ at singular points.

This would produce the behaviour at infinity which does not match with our instanton result (5.9).

Comparing coefficients in front of $1/m^4$ falloff of $Y(m)$ at large m in the (5.23) and (5.9) we make a prediction

$$\frac{6c}{c_0^2} = J_1, \quad (5.24)$$

where J_1 is given by (5.7). Calculation of the constant c from loop graph (5.19) will provide a nontrivial test of our exact solution (5.23). Note, that to test the position of the singularity (5.13) we need a two-instanton calculation.

6 Conclusion

In this paper we studied higher derivatives terms in $N=2$ SUSY QCD. We obtained an asymptotic behaviour of these terms in weak coupling regions of the modular space using the instanton-induced vertex approach. Then we concentrated on a particular operator (5.5) on the Higgs branch. We found the exact solution (5.23) for this operator studying its singular behaviour near the values of mass (5.13) at which singularities on the Coulomb branch collide.

Our result for the exact solution of higher derivative operator (5.5) shows that the integrable structure in $N=2$ supersymmetric gauge theories persists beyond just the leading term in the momentum expansion of the effective action (the prepotential). It gives an example that certain other operators in this expansion can be described in terms of analytic functions and found exactly.

Finding higher derivatives terms in the effective Lagrangian is important from physical point of view. They contribute to processes at non-zero energies as well as determine the dynamics of massive states. Moreover, if we want to deform the $N=2$ gauge theory to some QCD-like theory we ultimately have to take into account higher derivative terms [33].

As we explained in the previous section the general idea to study higher derivative terms is that their singularities are related to singularities of the prepotential. They come from the same particles becoming massless.

On the other hand we can use this correspondence in the opposite direction to extract some additional information about the dynamics of the

theory. The example of this is our relation (5.24). Knowing J_1 from instanton calculation we can use (5.24) to determine the constant c . This gives us coupling constant in the effective theory with mutually non-local light states. Another possibility of this kind is that singularities of certain higher derivative terms could correspond to electrically and magnetically neutral particles becoming massless. These singularities do not show up as a singularities of the prepotential. Study of higher derivative terms could give us some information about the existence of such massless states.

When this work was completed the author became aware of ref. [34] in which instanton-induced effective vertex was suggested for $N=2$ SUSY QCD and renormalization group flow to pure Yang-Mills theory was demonstrated (cf. our discussion in the beginning of Sect. 4). Authors of [34] use the $N=1$ superfield formulation and their result is in agreement with our eq. (3.5) for instanton-induced vertex in $N=1$ superfields. However, we have shown in Section 3 that $N=2$ supersymmetry requires extra terms to appear both in the exponential and in the measure. These terms make our final result (3.24) for the instanton-induced vertex different from that in [34].

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